

GRAVITATIONAL MICROLENSING RESEARCH

PHILIPPE JETZER

Paul Scherrer Institute, Laboratory for Astrophysics, CH-5232 Villigen PSI, and Institute of Theoretical Physics, University of Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland

One of the most important problems in astrophysics concerns the nature of the dark matter in galactic halos, whose presence is implied mainly by the observed flat rotation curves in spiral galaxies. In the framework of a baryonic scenario the most plausible candidates are brown dwarfs, M-dwarfs or white dwarfs and cold molecular clouds (mainly of H_2). The former can be detected with the ongoing microlensing experiments, which are rapidly leading to important new results. The French collaboration EROS and the American-Australian collaboration MACHO have reported until August 1997 the observation of ~ 16 microlensing events by monitoring during several years the brightness of millions of stars in the Large Magellanic Cloud and one event towards the Small Magellanic Cloud. In particular, the MACHO team found 8 microlensing candidates by analysing their first 2 years of observations. This implies that the halo dark matter fraction in form of MACHOs (Massive Astrophysical Compact Halo Objects) is of the order of 45-50% assuming a standard spherical halo model. More than 150 microlensing events have been detected in the direction of the galactic bulge by the MACHO, OGLE and DUO teams. The measured optical depth implies the presence of a bar in the galactic centre. Here, we give an overview of microlensing and the main results achieved so far.

1 Introduction

One of the most important problems in astrophysics concerns the nature of the dark matter in galactic halos, whose presence is implied by the observed flat rotation curves in spiral galaxies^{1,2}, the X-ray diffuse emission in elliptical galaxies as well as by the dynamics of galaxy clusters. Primordial nucleosynthesis entails that most of the baryonic matter in the Universe is nonluminous, and such an amount of dark matter falls suspiciously close to that required by the rotation curves. Surely, the standard model of elementary particle forces can hardly be viewed as the ultimate theory and all the attempts in that direction invariably call for new particles. Hence, the idea of nonbaryonic dark matter naturally enters the realm of cosmology and may help in the understanding of the process of galaxy formation and clustering of galaxies.

The problem of dark matter started already with the pioneering work of Oort³ in 1932 and Zwicky⁴ in 1933 and its mystery is still not solved. Actually, there are several dark matter problems on different scales ranging from the solar neighbourhood, galactic halos, cluster of galaxies to cosmological scales. Dark matter is also needed to understand the formation of large scale structures in the universe. Many candidates have been proposed, either baryonic or not, to explain dark matter.

Here, we discuss the dark matter problem in the halo of our Galaxy in connection with microlensing searches. We present the basics of microlensing and an overview of the results obtained so far, without, however, being exhaustive and this also for the references we quote. The content is as follows: first, we review the evidence for dark matter in the halo of our Galaxy. In Section 3 we present the baryonic candidates for dark matter and in Section 4 we give an overview of the

results of microlensing searches achieved so far. In Section 5 we discuss the basics of microlensing (optical depth, microlensing rates, etc.) and in Section 6 we briefly present a scenario in which part of the dark matter is in the form of cold molecular clouds (mainly of H_2).

2 Mass of the Milky Way

The best evidence for dark matter in galaxies comes from the rotation curves of spirals. Measurements of the rotation velocity v_{rot} of stars up to the visible edge of the spiral galaxies and of HI gas in the disk beyond the optical radius (by measuring the Doppler shift in the 21-cm line) imply that $v_{rot} \approx$ constant out to very large distances, rather than to show a Keplerian falloff. These observations started around 1970⁵, thanks to the improved sensitivity in both optical and 21-cm bands. By now there are observations for over thousand spiral galaxies with reliable rotation curves out to large radii⁶. In almost all of them the rotation curve is flat or slowly rising out to the last measured point. Very few galaxies show falling rotation curves and those that do either fall less rapidly than Keplerian have nearby companions that may perturb the velocity field or have large spheroids that may increase the rotation velocity near the centre.

There are also measurements of the rotation velocity for our Galaxy. However, these observations turn out to be rather difficult, and the rotation curve has been measured only up to a distance of about 20 kpc. Without any doubt our own galaxy has a typical flat rotation curve. A fact this, which implies that it is possible to search directly for dark matter characteristic of spiral galaxies in our own Milky Way.

In order to infer the total mass one can also study the proper motion of the Magellanic Clouds and of other satellites of our Galaxy. Recent studies^{7,8,9} do not yet allow an accurate determination of $v_{rot}(LMC)/v_0$ ($v_0 = 210 \pm 10$ km/s being the local rotational velocity). Lin et al.⁸ analyzed the proper motion observations and concluded that within 100 kpc the Galactic halo has a mass $\sim 5.5 \pm 1 \times 10^{11} M_\odot$ and a substantial fraction $\sim 50\%$ of this mass is distributed beyond the present distance of the Magellanic Clouds of about 50 kpc. Beyond 100 kpc the mass may continue to increase to $\sim 10^{12} M_\odot$ within its tidal radius of about 300 kpc. This value for the total mass of the Galaxy is in agreement with the results of Zaritsky et al.⁷, who found a total mass in the range 9.3 to $12.5 \times 10^{11} M_\odot$, the former value by assuming radial satellite orbits whereas the latter by assuming isotropic satellite orbits.

The results of Lin et al.⁸ suggest that the mass of the halo dark matter up to the Large Magellanic Cloud (LMC) is roughly half of the value one gets for the standard halo model (with flat rotation curve up to the LMC and spherical shape), implying thus the same reduction for the number of expected microlensing events. Kochanek⁹ analysed the global mass distribution of the Galaxy adopting a Jaffe model, whose parameters are determined using the observations on the proper motion of the satellites of the Galaxy, the Local Group timing constraint and the ellipticity of the M31 orbit. With these observations Kochanek⁹ concludes that the mass inside 50 kpc is $5.4 \pm 1.3 \times 10^{11} M_\odot$. This value becomes, however, slightly smaller when using only the satellite observations and the disk rotation constraint,

in this case the median mass interior to 50 kpc is in the interval 3.3 to 6.1 (4.2 to 6.8) without (with) Leo I satellite in units of $10^{11} M_{\odot}$. The lower bound without Leo I is 65% of the mass expected assuming a flat rotation curve up to the LMC.

3 Baryonic dark matter candidates

Before discussing the baryonic dark matter we would like to mention that another class of candidates which is seriously taken into consideration is the so-called cold dark matter, which consists for instance of axions or supersymmetric particles like neutralinos¹⁰. Here, we will not discuss cold dark matter in detail. However, recent studies seem to point out that there is a discrepancy between the calculated (through N-body simulations) rotation curve for dwarf galaxies assuming an halo of cold dark matter and the measured curves^{11,12,13}. If this fact is confirmed, this would exclude cold dark matter as a major constituent of the halo of dwarf galaxies and possibly also of spiral galaxies.

From the Big Bang nucleosynthesis model^{14,15} and from the observed abundances of primordial elements one infers: $0.010 \leq h_0^2 \Omega_B \leq 0.016$ or with $h_0 \simeq 0.4 - 1$ one gets $0.01 \leq \Omega_B \leq 0.10$ (where $\Omega_B = \rho_B / \rho_{crit}$, and $\rho_{crit} = 3H_0^2 / 8\pi G$). Since for the amount of luminous baryons one finds $\Omega_{lum} \ll \Omega_B$, it follows that an important fraction of the baryons are dark. In fact the dark baryons may well make up the entire dark halo matter.

The halo dark matter cannot be in the form of hot ionized hydrogen gas otherwise there would be a large X-ray flux, for which there are stringent upper limits¹⁶. The abundance of neutral hydrogen gas is inferred from the 21-cm measurements, which show that its contribution is small. Another possibility is that the hydrogen gas is in molecular form clumped into cold clouds, as we will briefly discuss in Section 6. Baryons could otherwise have been processed in stellar remnants (for a detailed discussion see¹⁷). If their mass is below $\sim 0.08 M_{\odot}$ they are too light to ignite hydrogen burning reactions. The possible origin of such brown dwarfs or Jupiter like bodies (called also MACHOs), by fragmentation or by some other mechanism, is at present not well understood. It has also been pointed out that the mass distribution of the MACHOs, normalized to the dark halo mass density, could be a smooth continuation of the known initial mass function of ordinary stars¹⁸. The ambient radiation, or their own body heat, would make sufficiently small objects of H and He evaporate rapidly. The condition that the rate of evaporation of such a hydrogenoid sphere be insufficient to halve its mass in a billion years leads to the following lower limit on their mass¹⁸: $M > 10^{-7} M_{\odot} (T_S / 30 K)^{3/2} (1 g cm^{-3} / \rho)^{1/2}$ (T_S being their surface temperature and ρ their average density, which we expect to be of the order $\sim 1 g cm^{-3}$).

Otherwise, MACHOs might be either M-dwarfs or else white dwarfs. As a matter of fact, a deeper analysis shows that the M-dwarf option looks problematic. The null result of several searches for low-mass stars both in the disk and in the halo of our Galaxy suggests that the halo cannot be mostly in the form of hydrogen burning main sequence M-dwarfs. Optical imaging of high-latitude fields taken with the Wide Field Camera of the Hubble Space Telescope indicates that less than $\sim 6\%$ of the halo can be in this form¹⁹. Observe, however, that these results are derived

under the assumption of a smooth spatial distribution of M-dwarfs, and become considerably less severe in the case of a clumpy distribution^{20,21}.

A scenario with white dwarfs as a major constituent of the galactic halo dark matter has been explored²². However, it requires a rather ad hoc initial mass function sharply peaked around $2 - 6 M_{\odot}$. Future Hubble deep field exposures could either find the white dwarfs or put constraints on their fraction in the halo²³. Also a substantial component of neutron stars and black holes with mass higher than $\sim 1 M_{\odot}$ is excluded, for otherwise they would lead to an overproduction of heavy elements relative to the observed abundances.

4 Present status of microlensing research

It has been pointed out by Paczyński²⁴ that microlensing allows the detection of MACHOs located in the galactic halo in the mass range¹⁸ $10^{-7} < M/M_{\odot} < 1$, as well as MACHOs in the disk or bulge of our Galaxy^{25,26}. Since this first proposal microlensing searches have turned very quickly into reality and in about a decade they have become an important tool for astrophysical investigations. Microlensing is very promising for the search of planets around other stars in our Galaxy and generates also very large databases for variable stars, a field which has already benefitted a lot. Because of the increase of observations, due also to the fact that new experiments are becoming operative, the situation is changing rapidly and, therefore, the present results should be considered as preliminary. Within few years the amount of data will be such that several problems will be solved or at least allow to achieve substantial progress. The following presentation is also not exhaustive with respect to all what has been found so far.

4.1 Towards the LMC and the SMC

In September 1993 the French collaboration EROS²⁷ announced the discovery of 2 microlensing candidates and the American–Australian collaboration MACHO of one candidate²⁸ by monitoring stars in the LMC.

In the meantime the MACHO team reported the observation of altogether 8 events (one is a binary lensing event) analysing their first two years of data by monitoring about 8.5 million of stars in the LMC²⁹. The inferred optical depth is $\tau = 2.1_{-0.7}^{+1.1} \times 10^{-7}$ when considering 6 events^a (see Table 3) (or $\tau = 2.9_{-0.9}^{+1.4} \times 10^{-7}$ when considering all the 8 detected events). Correspondingly, this implies that about 45% (50% respectively) of the halo dark matter is in form of MACHOs and they find an average mass $0.5_{-0.2}^{+0.3} M_{\odot}$ assuming a standard spherical halo model. It may well be that there is also a contribution of events due to MACHOs located in the LMC itself or in a thick disk of our galaxy, the corresponding optical depth is estimated to be only²⁹ $\tau = 5.4 \times 10^{-8}$. Other events have been detected towards the LMC by the MACHO group, which have been put on their list of alert events. In particular 2 events are reported in 1996 and already 4 events in 1997. The full analysis of the 1996 and 1997 seasons is still not published.

^aIn fact, the two disregarded events are a binary lensing and one which is rated as marginal.

EROS has also searched for very-low mass MACHOs by looking for microlensing events with time scales ranging from 30 minutes to 7 days³⁰. The lack of candidates in this range places significant constraints on any model for the halo that relies on objects in the range $5 \times 10^{-8} < M/M_{\odot} < 2 \times 10^{-2}$. Indeed, such objects may make up at most 20% of the halo dark matter (in the range between $5 \times 10^{-7} < M/M_{\odot} < 2 \times 10^{-3}$ at most 10%). Similar conclusions have also been reached by the MACHO group²⁹. Recently, the MACHO team reported³¹ the first discovery of a microlensing event towards the Small Magellanic Cloud (SMC). The full analysis of the four years data on the SMC is still underway, so that more candidates may be found in the near future. A rough estimate of the optical depth leads to about the same value as found towards the LMC.

The EROS group has completed his first campaign based on photographic plates and CCD, the latter one for the short duration events. Since the middle of 1996 the EROS group has put into operation a new 1 meter telescope, located in La Silla (Chile), and which is fully dedicated to microlensing searches using CCD cameras. The improved experiment is called EROS II.

4.2 Towards the galactic centre

Towards the galactic bulge the Polish-American team OGLE³² announced his first event also in September 1993. Since then OGLE found in their data from the 1992 - 1995 observing seasons altogether 18 microlensing events (one being a binary lens). Based on their first 9 events the OGLE team estimated the optical depth towards the bulge as³³ $\tau = (3.3 \pm 1.2) \times 10^{-6}$. This has to be compared with the theoretical calculations which lead to a value^{25,26} $\tau \simeq (1 - 1.5) \times 10^{-6}$, which does, however, not take into account the contribution of lenses in the bulge itself, which might well explain the discrepancy. In fact, when taking into account also the effect of microlensing by galactic bulge stars the optical depth gets bigger³⁴ and might easily be compatible with the measured value. This implies the presence of a bar in the galactic centre. In the meantime the OGLE group got a new dedicated 1.3 meter telescope located at the Las Campanas Observatory. The OGLE-2 collaboration has started the observations in 1996 and is monitoring the bulge, the LMC and the SMC as well.

The French DUO³⁵ team found 12 microlensing events (one of which being a binary event) by monitoring the galactic bulge during the 1994 season with the ESO 1 meter Schmidt telescope. The photographic plates were taken in two different colors to test achromaticity. The MACHO³⁶ collaboration found by now more than ~ 150 microlensing events towards the galactic bulge, most of which are listed among the alert events, which are constantly updated^b. They found also 3 events by monitoring the spiral arms in the region of Gamma Scutum. During their first season they found 45 events towards the bulge. The MACHO team detected also in a long duration event the parallax effect due to the motion of the Earth around the Sun³⁷. The MACHO first year data leads to an estimated optical depth of $\tau \simeq 2.43^{+0.54}_{-0.45} \times 10^{-6}$, which is roughly in agreement with the OGLE result, and

^bCurrent information on the MACHO Collaboration's Alert events is maintained at the WWW site <http://darkstar.astro.washington.edu>.

which also implies the presence of a bar in the galactic centre. These results are very important in order to study the structure of our Galaxy. In this respect the measurement towards the spiral arms will give important new information.

4.3 Towards the Andromeda galaxy

Microlensing searches have also been conducted towards M31. In this case, however, one has to use the so-called “pixel-lensing” method, since the source stars are in general no longer resolvable. Two groups have performed searches: the French AGAPE ³⁸ using the 2 meter telescope at Pic du Midi and the American VATT/COLUMBIA ³⁹, which used the 1.8 meter VATT-telescope located on Mt. Graham and the 4 meter KNPO telescope. Both teams showed that the pixel-lensing method works, however, the small amount of observations done so far does not allow to draw firm conclusions. The VATT/COLUMBIA team found six candidates which are consistent with microlensing, however, additional observations are needed to confirm this. Pixel-lensing could also lead to the discovery of microlensing events towards the M87 galaxy, in which case the best would be to use the Hubble Space Telescope ⁴⁰. It might also be interesting to look towards dwarf galaxies of the local group.

4.4 Further developments

A new collaboration between New Zealand and Japan, called MOA, started in june 1996 to perform observations using the 0.6 meter telescope of the Mt. John Observatory ⁴¹. The targets are the LMC and the galactic bulge. They will in particular search for short timescale (~ 1 hour) events, and would then be particularly sensitive to objects with a mass typical for brown dwarfs.

It has to be mentioned that there are also collaborations between different observatories (for instance PLANET ⁴² and GMAN ⁴³) with the aim to perform accurate photometry on alert microlensing events. The GMAN collaboration was able to accurately get photometric data on a 1995 event towards the galactic bulge. The light curve shows clearly a deviation due to the extension of the source star ⁴⁴. A major goal of the PLANET and GMAN collaborations is to find planets in binary microlensing events ^{45,46,47}. Moreover, microlensing searches are also very powerful ways to get large database for the study and discovery of many variable stars.

At present the only information available from a microlensing event is the time scale, which depends on three parameters: distance, transverse velocity and mass of the MACHO. A possible way to get more information is to observe an event from different locations, with typically an Astronomical Unit in separation. This could be achieved by putting a parallax satellite into solar orbit ^{48,49}.

The above list of presently active collaborations and main results shows clearly that this field is just at the beginning and that many interesting results will come in the near future.

5 Basics of microlensing

In the following we present the main features of microlensing, in particular its probability and rate of events (for reviews see also ^{50,51}, whereas for double lenses see

for instance ref. ⁵²). An important issue is the determination from the observations of the mass of the MACHOs that acted as gravitational lenses as well as the fraction of halo dark matter they make up. The most appropriate way to compute the average mass and other important information is to use the method of mass moments developed by De Rújula et al. ⁵³, which will be briefly discussed in Section 5.6.

5.1 Microlensing probability

When a MACHO of mass M is sufficiently close to the line of sight between us and a more distant star, the light from the source suffers a gravitational deflection. The deflection angle is usually so small that we do not see two images but rather a magnification of the original star brightness. This magnification, at its maximum, is given by

$$A_{max} = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}. \quad (1)$$

Here $u = d/R_E$ (d is the distance of the MACHO from the line of sight) and the Einstein radius R_E is defined as

$$R_E^2 = \frac{4GMD}{c^2}x(1-x) \quad (2)$$

with $x = s/D$, and where D and s are the distance between the source, respectively the MACHO and the observer.

An important quantity is the optical depth τ_{opt} to gravitational microlensing defined as

$$\tau_{opt} = \int_0^1 dx \frac{4\pi G}{c^2} \rho(x) D^2 x(1-x) \quad (3)$$

with $\rho(x)$ the mass density of microlensing matter at distance $s = xD$ from us along the line of sight. The quantity τ_{opt} is the probability that a source is found within a radius R_E of some MACHO and thus has a magnification that is larger than $A = 1.34$ ($d \leq R_E$).

We calculate τ_{opt} for a galactic mass distribution of the form

$$\rho(\vec{r}) = \frac{\rho_0(a^2 + R_{GC}^2)}{a^2 + \vec{r}^2}, \quad (4)$$

$|\vec{r}|$ being the distance from the Earth. Here, a is the core radius, ρ_0 the local dark mass density in the solar system and R_{GC} the distance between the observer and the Galactic centre. Standard values for the parameters are $\rho_0 = 0.3 \text{ GeV}/\text{cm}^3 = 7.9 \cdot 10^{-3} M_\odot/\text{pc}^3$, $a = 5.6 \text{ kpc}$ and $R_{GC} = 8.5 \text{ kpc}$. With these values we get, for a spherical halo, $\tau_{opt} \simeq 5 \times 10^{-7}$ for the LMC and $\tau_{opt} \simeq 7 \times 10^{-7}$ for the SMC ⁵⁴.

The magnification of the brightness of a star by a MACHO is a time-dependent effect. For a source that can be considered as pointlike (this is the case if the projected star radius at the MACHO distance is much less than R_E) the light curve as a function of time is obtained by inserting

$$u(t) = \frac{(d^2 + v_T^2 t^2)^{1/2}}{R_E} \quad (5)$$

into eq.(1), where v_T is the transverse velocity of the MACHO, which can be inferred from the measured rotation curve ($v_T \approx 200 \text{ km/s}$). The achromaticity, symmetry and uniqueness of the signal are distinctive features that allow to discriminate a microlensing event from background events such as variable stars.

The behaviour of the magnification with time, $A(t)$, determines two observables namely, the magnification at the peak $A(0)$ - denoted by A_{max} - and the width of the signal T (defined as being $T = R_E/v_T$).

5.2 Microlensing rate towards the LMC

The microlensing rate depends on the mass and velocity distribution of MACHOs. The mass density at a distance $s = xD$ from the observer is given by eq.(4). The isothermal spherical halo model does not determine the MACHO number density as a function of mass. A simplifying assumption is to let the mass distribution be independent of the position in the galactic halo, i.e., we assume the following factorized form for the number density per unit mass dn/dM ,

$$\frac{dn}{dM} dM = \frac{dn_0}{d\mu} \frac{a^2 + R_{GC}^2}{a^2 + R_{GC}^2 + D^2 x^2 - 2DR_{GC} x \cos\alpha} d\mu = \frac{dn_0}{d\mu} H(x) d\mu, \quad (6)$$

with $\mu = M/M_\odot$ (α is the angle of the line of sight with the direction of the galactic centre, which is 82^0 for the LMC), n_0 not depending on x and is subject to the normalization $\int d\mu \frac{dn_0}{d\mu} M = \rho_0$. Nothing a priori is known on the distribution dn_0/dM .

A different situation arises for the velocity distribution in the isothermal spherical halo model, its projection in the plane perpendicular to the line of sight leads to the following distribution in the transverse velocity v_T

$$f(v_T) = \frac{2}{v_H^2} v_T e^{-v_T^2/v_H^2}. \quad (7)$$

($v_H \approx 210 \text{ km/s}$ is the observed velocity dispersion in the halo).

In order to find the rate at which a single star is microlensed with magnification $A \geq A_{min}$, we consider MACHOs with masses between M and $M + \delta M$, located at a distance from the observer between s and $s + \delta s$ and with transverse velocity between v_T and $v_T + \delta v_T$. The collision time can be calculated using the well-known fact that the inverse of the collision time is the product of the MACHO number density, the microlensing cross-section and the velocity. The rate $d\Gamma$, taken also as a differential with respect to the variable u , at which a single star is microlensed in the interval $d\mu dudv_T dx$ is given by^{53,55}

$$d\Gamma = 2v_T f(v_T) D r_E [\mu x(1-x)]^{1/2} H(x) \frac{dn_0}{d\mu} d\mu dudv_T dx, \quad (8)$$

with

$$r_E^2 = \frac{4GM_\odot D}{c^2} \sim (3.2 \times 10^9 \text{ km})^2. \quad (9)$$

One has to integrate the differential number of microlensing events, $dN_{ev} = N_\star t_{obs} d\Gamma$, over an appropriate range for μ , x , u and v_T , in order to obtain the total

number of microlensing events which can be compared with an experiment monitoring N_\star stars during an observation time t_{obs} and which is able to detect a magnification such that $A_{max} \geq A_{TH}$. The limits of the u integration are determined by the experimental threshold in magnitude shift, Δm_{TH} : we have $0 \leq u \leq u_{TH}$.

The range of integration for μ is where the mass distribution $dn_0/d\mu$ is not vanishing and that for x is $0 \leq x \leq D_h/D$ where D_h is the extent of the galactic halo along the line of sight (in the case of the LMC, the star is inside the galactic halo and thus $D_h/D = 1$.) The galactic velocity distribution is cut at the escape velocity $v_e \approx 640 \text{ km/s}$ and therefore v_T ranges over $0 \leq v_T \leq v_e$. In order to simplify the integration we integrate v_T over all the positive axis, due to the exponential factor in $f(v_T)$ the so committed error is negligible.

However, the integration range of $d\mu dudv_T dx$ does not span all the interval we have just described. Indeed, each experiment has time thresholds T_{min} and T_{max} and only detects events with: $T_{min} \leq T \leq T_{max}$, and thus the integration range has to be such that T lies in this interval. The total number of micro-lensing events is then given by

$$N_{ev} = \int dN_{ev} \epsilon(T) , \quad (10)$$

where the integration is over the full range of $d\mu dudv_T dx$. $\epsilon(T)$ is determined experimentally^{29,36} or if not known can, for instance, be taken as follows $\epsilon(T) = \Theta(T - T_{min})\Theta(T_{max} - T)$. T is related in a complicated way to the integration variables, because of this, no direct analytical integration in eq.(10) can be performed.

To evaluate eq.(10) we define an efficiency function $\epsilon_0(\mu)$

$$\epsilon_0(\mu) \equiv \frac{\int dN_{ev}^\star(\bar{\mu}) \epsilon(T)}{\int dN_{ev}^\star(\bar{\mu})} , \quad (11)$$

which measures the fraction of the total number of microlensing events that meet the condition on T at a fixed MACHO mass $M = \bar{\mu}M_\odot$. We now can write the total number of events in eq.(10) as

$$N_{ev} = \int dN_{ev} \epsilon_0(\mu) . \quad (12)$$

Due to the fact that ϵ_0 is a function of μ alone, the integration in $d\mu dudv_T dx$ factorizes into four integrals with independent integration limits.

The average lensing duration can be defined as follows

$$\langle T \rangle = \frac{1}{\Gamma} \int d\Gamma T(x, \mu, v_T) , \quad (13)$$

where $T(x, \mu, v_T) = R_E(x, \mu)/v_T$. One easily finds that $\langle T \rangle$ satisfies the following relation

$$\langle T \rangle = \frac{2\tau}{\pi\Gamma} u_{TH} . \quad (14)$$

In order to quantify the expected number of events it is convenient to take as an example a delta function distribution for the mass. The rate of microlensing

events with $A \geq A_{min}$ (or $u \leq u_{max}$), is then

$$\Gamma(A_{min}) = u_{max} \tilde{\Gamma} = u_{max} D r_E \sqrt{\pi} v_H \frac{\rho_0}{M_\odot} \frac{1}{\sqrt{\mu}} \int_0^1 dx [x(1-x)]^{1/2} H(x) . \quad (15)$$

Inserting the numerical values for the LMC (D=50 kpc and $\alpha = 82^0$) we get

$$\tilde{\Gamma} = 4 \times 10^{-13} \left(\frac{v_H}{210 \text{ km/s}} \right) \left(\frac{\rho_0}{0.3 \text{ GeV/cm}^3} \right) \frac{1}{\sqrt{M/M_\odot}} \text{ s}^{-1} . \quad (16)$$

For an experiment monitoring N_\star stars during an observation time t_{obs} the total number of events with a magnification $A \geq A_{min}$ is: $N_{ev}(A_{min}) = N_\star t_{obs} \Gamma(A_{min})$. In the following Table 1 we show some values of N_{ev} for the LMC, taking $t_{obs} = 1$ year, $N_\star = 10^6$ stars and $A_{min} = 1.34$ (or $\Delta m_{min} = 0.32$).

Table 1

MACHO mass in M_\odot	Mean R_E in km	Mean microlensing time	N_{ev}
10^{-1}	0.3×10^9	1 month	4.5
10^{-2}	10^8	9 days	15
10^{-4}	10^7	1 day	165
10^{-6}	10^6	2 hours	1662

As mentioned in Sect. 4.1 the MACHO team found (till September 1997) altogether 14 events (one of which being a binary lens) towards the LMC and one event towards the SMC. The EROS team found 2 events using photographic plate techniques.

5.3 Microlensing rate towards M31

Gravitational microlensing is also useful for detecting MACHOs in the halo of nearby galaxies^{56,57} such as M31 or M33, for which the experiments AGAPE and VATT/Columbia are under way. In fact, it turns out that the massive dark halo of M31 has an optical depth to microlensing which is of about the same order of magnitude as that of our own galaxy^{56,57,58} $\sim 10^{-6}$. Moreover, an experiment monitoring stars in M31 is sensitive to both MACHOs in our halo and in the one of M31. One can also compute the microlensing rate⁵⁸ for MACHOs in the halo of M31, for which we get

$$\tilde{\Gamma} = 1.8 \times 10^{-12} \left(\frac{v_H}{210 \text{ km/s}} \right) \left(\frac{\rho(0)}{1 \text{ GeV/cm}^3} \right) \frac{1}{\sqrt{M/M_\odot}} \text{ s}^{-1} . \quad (17)$$

($\rho(0)$ is the central density of dark matter.) The average lensing time is given by

$$\langle T \rangle \sim (125 \text{ days}) \sqrt{M/M_\odot} . \quad (18)$$

In the following Table 2 we show some values of N_{ev}^a due to MACHOs in the halo of M31 with $t_{obs} = 1$ year and $N_\star = 10^6$ stars. In the last column we give the corresponding number of events, N_{ev} , due to MACHOs in our own halo. The mean microlensing time is about the same for both types of events.

Table 2

MACHO mass in M_{\odot}	Mean R_E in km	Mean microlensing time	N_{ev}^a	N_{ev}
10^{-1}	7×10^8	38 days	6	3
10^{-2}	2×10^8	12 days	21	12
10^{-4}	2×10^7	30 hours	210	129
10^{-6}	2×10^6	3 hours	2100	1290

N_{ev}^a is almost by a factor of two bigger than N_{ev} (see also⁵⁸). Of course these numbers should be taken as an estimate, since they depend on the details of the model one adopts for the distribution of the dark matter in the halo. To distinguish between events due to a MACHO in our halo or in the one of M31 might be rather difficult.

5.4 Microlensing rate towards the galactic bulge

We compute the microlensing rate Γ for an experiment monitoring stars in the galactic bulge in Baade's window of galactic coordinates (longitude and latitude): $l = 1^\circ$, $b = -3.^\circ 9$. $D = 8.5$ kpc is the distance to stars in the galactic bulge, $r_E = 1.25 \times 10^9$ km and we use the definition $T = R_E/v_T$.

The number density of disk stars per unit mass is given by⁵⁹

$$\frac{dn}{dM} = \frac{dn_0}{dM} \exp\left(-\frac{Dx |\sin b|}{300 \text{ pc}} + \frac{Dx \cos b}{3.5 \text{ kpc}}\right) = \frac{dn_0}{dM} H_d(x) , \quad (19)$$

where the galactic longitude $l = 0^\circ$ has been adopted and $\frac{dn_0}{dM} = \frac{\rho_d(M)}{M}$ with $\rho_d(M) = 0.05 M_{\odot} \text{ pc}^{-3}$; H_d can be written as follows

$$H_d(x) = \exp\left(\frac{xD (1 - 11.7 |\sin b|)}{3.5 \text{ kpc}}\right) , \quad (20)$$

using also the fact that $\cos b \approx 1$ for the galactic bulge.

For the contribution $H(x)$ of the halo dark matter in form of MACHOs located in the disk one uses the distribution for the number density per unit mass dn/dM as given in eq. (6) with $\alpha \simeq 4^\circ$ (the angle between the line of sight and the direction of the galactic centre).

With the above distribution for the lenses in the disk it follows that the corresponding optical depth is $\tau \simeq 7 \times 10^{-7}$ (and $\tau \simeq 1.2 \times 10^{-7}$ for the halo contribution).

In computing Γ one must also take into account the fact that both the source and the observer are in motion⁵⁵. Relevant are only the velocities transverse to the line of sight. The transverse velocity of the microlensing tube at position xD is: $\vec{v}_t(x) = (1 - x)\vec{v}_{\odot\perp} + x\vec{v}_{s\perp}$, and its magnitude is

$$v_t(x) = \sqrt{(1 - x)^2 |\vec{v}_{\odot\perp}|^2 + x^2 |\vec{v}_{s\perp}|^2 + 2x(1 - x) |\vec{v}_{\odot\perp}| |\vec{v}_{s\perp}| \cos \theta} , \quad (21)$$

where $\vec{v}_{s\perp}$ and $\vec{v}_{\odot\perp}$ are the source and the solar velocities transverse to the line of sight and θ the angle between them.

For the velocity distribution of the MACHOs or the faint disk stars we consider an isothermal spherical model, which in the rest frame of the galaxy is given by

$$f(\vec{v})d^3v = \frac{1}{\tilde{v}_H^3 \pi^{3/2}} e^{-\vec{v}^2/\tilde{v}_H^2} d^3v. \quad (22)$$

Since only the transverse velocities are of relevance cylindrical coordinates can be used and the integration made over the velocity component parallel to the line of sight. Moreover, due to the velocities of the observer and the source, the value of the transverse velocity gets shifted by $\vec{v}_t(x)$. The distribution for the transverse velocity is thus

$$\tilde{f}(v_T)dv_T = \frac{1}{\pi v_H^2} e^{-(\vec{v}_T - \vec{v}_t)^2/v_H^2} v_T dv_T, \quad (23)$$

where $v_H \approx 30 \text{ km s}^{-1}$ is the velocity dispersion²⁵.

The random velocity of the source stars in the bulge are again described by an isothermal spherical distribution, whose transverse velocity distribution is

$$g(v_{s\perp})dv_{s\perp} = \frac{1}{\pi v_D^2} e^{-v_{s\perp}^2/v_D^2} v_{s\perp} dv_{s\perp}, \quad (24)$$

where $v_D = 156 \text{ km s}^{-1}$ is the velocity dispersion^{60,25,26}.

Taking all the above facts into account, Γ turns out to be⁶¹

$$\begin{aligned} \Gamma = & 4 r_E D v_H u_{TH} \left(\int_0^\infty \sqrt{\mu} \frac{dn_0}{d\mu} d\mu \right) \int_0^{2\pi} d\theta \int_0^\infty dv_{s\perp} g(v_{s\perp}) \\ & \int_0^1 \sqrt{x(1-x)} H_i(x) e^{-\eta^2} \int_0^\infty dy y^2 I_0(2y\eta) e^{-y^2}, \end{aligned} \quad (25)$$

where $y = \frac{v_T}{v_H}$, $\eta(x, \theta, v_{s\perp}, v_{\odot\perp}) = \frac{v_t}{v_H}$, and I_0 is the modified Bessel function of order 0. In the limit of stationary observer and source star ($v_{\odot\perp} = v_{s\perp} = 0$), $\eta = 0$ and $I_0 = 1$; H_i means either H_d or H ; u_{TH} is related to the minimal experimentally detectable magnification $A_{TH} = A[u = u_{TH}]$; $v_{\odot\perp}$ is $\approx 220 \text{ km s}^{-1}$ (more precisely it should be multiplied by $\cos l$, where l is the galactic longitude but since $l = 1^\circ$, $\cos l \approx 1$). In computing Γ one should also take into account the limited measurable range for the event duration T , which translates into a modification of the integration limits. A fact that can be described by introducing an efficiency function⁵³ $\epsilon_0(\mu)$.

Assuming a delta-function-type distribution for the masses

$$\frac{dn_0}{d\mu} = \frac{\rho}{M_\odot} \frac{\delta(\mu - \bar{\mu})}{\mu}, \quad (26)$$

eq. (25) can be integrated. With $N_\star = 10^6$ stars and $t_{obs} = 1$ year one gets

$$N_{ev} = \frac{2.08}{\sqrt{\mu}} \left(\frac{\rho_d}{5 \times 10^{-2} M_\odot \text{ pc}^{-3}} \right) u_{TH}, \quad (27)$$

for $H_i = H_d$, and

$$N_{ev} = \frac{0.61}{\sqrt{\mu}} \left(\frac{\rho_0}{8 \times 10^{-3} M_\odot \text{ pc}^{-3}} \right) u_{TH}, \quad (28)$$

for $H_i = H$. The numerical factor in eq. (27) for N_{ev} as a function of b , the galactic latitude, varies between 6.8 for $b = 0^\circ$ and 1.9 for $b = 5^\circ$, whereas the factor for N_{ev} of eq. (28) remains practically unchanged.

As mentioned in Section 4.2 the results clearly show that there is a bar in the galactic centre and that one has to consider also the bar-bar (or bulge-bulge) contribution in order to explain the observations.

5.5 Most probable mass for a single event

The probability P that a microlensing event of duration T and maximum amplification A_{max} be produced by a MACHO of mass μ (in units of M_\odot) is given by⁶²

$$P(\mu, T) \propto \frac{\mu^2}{T^4} \int_0^1 dx (x(1-x))^2 H(x) \exp\left(-\frac{r_E^2 \mu x(1-x)}{v_H^2 T^2}\right), \quad (29)$$

which does not depend on A_{max} and $P(\mu, T) = P(\mu/T^2)$. The measured values for T towards the LMC are listed in Table 3, where μ_{MP} is the most probable value. The normalization is arbitrarily chosen such that the maximum of $P(\mu_{MP}, T) = 1$. We find that the maximum corresponds to $\mu r_E^2/v_H^2 T^2 = 13.0$ ^{62,61}. The 50% confidence interval embraces for the mass μ approximately the range $1/3\mu_{MP}$ up to $3\mu_{MP}$. Similarly one can compute $P(\mu, T)$ also for the bulge events⁶¹.

Table 3: Values of μ_{MP} (in M_\odot) for eight microlensing events detected in the LMC (A_i = American-Australian collaboration events ($i = 1, \dots, 6$); F_1 and F_2 French collaboration events). For the LMC: $v_H = 210 \text{ km s}^{-1}$ and $r_E = 3.17 \times 10^9 \text{ km}$.

	A_1	A_2	A_3	A_4	A_5	A_6	F_1	F_2
T (days)	17.3	23	31	41	43.5	57.5	27	30
$\tau (\equiv \frac{v_H}{r_E} T)$	0.099	0.132	0.177	0.235	0.249	0.329	0.155	0.172
μ_{MP}	0.13	0.23	0.41	0.72	0.81	1.41	0.31	0.38

5.6 Mass moment method

A more systematic way to extract information on the masses is to use the method of mass moments⁵³. The mass moments $\langle \mu^m \rangle$ are defined as

$$\langle \mu^m \rangle = \int d\mu \epsilon_n(\mu) \frac{dn_0}{d\mu} \mu^m. \quad (30)$$

$\langle \mu^m \rangle$ is related to $\langle \tau^n \rangle = \sum_{events} \tau^n$, with $\tau \equiv (v_H/r_E)T$, as constructed from the observations and which can also be computed as follows

$$\langle \tau^n \rangle = \int dN_{ev} \epsilon_n(\mu) \tau^n = V u_{TH} \gamma(m) \langle \mu^m \rangle, \quad (31)$$

with $m \equiv (n+1)/2$. For targets in the LMC $\gamma(m) = \Gamma(2-m)\hat{H}(m)$ and

$$V \equiv 2N_\star t_{obs} D r_E v_H = 2.4 \times 10^3 \text{ pc}^3 \frac{N_\star t_{obs}}{10^6 \text{ star-years}}, \quad (32)$$

$$\Gamma(2-m) \equiv \int_0^\infty \left(\frac{v_T}{v_H} \right)^{1-n} f(v_T) dv_T , \quad (33)$$

$$\hat{H}(m) \equiv \int_0^1 (x(1-x))^m H(x) dx . \quad (34)$$

The efficiency $\epsilon_n(\mu)$ is determined as follows (see⁵³)

$$\epsilon_n(\mu) \equiv \frac{\int dN_{ev}^*(\bar{\mu}) \epsilon(T) \tau^n}{\int dN_{ev}^*(\bar{\mu}) \tau^n} , \quad (35)$$

where $dN_{ev}^*(\bar{\mu})$ is defined as dN_{ev} in eq.(10) with the MACHO mass distribution concentrated at a fixed mass $\bar{\mu}$: $dn_0/d\mu = n_0 \delta(\mu - \bar{\mu})/\mu$. $\epsilon(T)$ is the experimental detection efficiency. For a more detailed discussion on the efficiency see ref.⁶³.

A mass moment $\langle \mu^m \rangle$ is thus related to $\langle \tau^n \rangle$ as given from the measured values of T in a microlensing experiment by

$$\langle \mu^m \rangle = \frac{\langle \tau^n \rangle}{Vu_{TH}\gamma(m)} . \quad (36)$$

The mean local density of MACHOs (number per cubic parsec) is $\langle \mu^0 \rangle$. The average local mass density in MACHOs is $\langle \mu^1 \rangle$ solar masses per cubic parsec. In the following we consider only 6 (see Table 3) out of the 8 events observed by the MACHO group, in fact the two events we neglect are a binary lensing event and an event which is rated as marginal. The mean mass, which we get from the six events detected by the MACHO team, is⁶⁴

$$\frac{\langle \mu^1 \rangle}{\langle \mu^0 \rangle} = 0.27 M_\odot . \quad (37)$$

(To obtain this result we used the values of τ as reported in Table 3, whereas $\Gamma(1)\hat{H}(1) = 0.0362$ and $\Gamma(2)\hat{H}(0) = 0.280$ as plotted in Fig. 6 of ref.⁵³). When taking for the duration T the values corrected for “blending”, we get as average mass $0.34 M_\odot$. If we include also the two EROS events we get a value of $0.26 M_\odot$ for the mean mass (without taking into account blending effects). The resulting mass depends on the parameters used to describe the standard halo model. In order to check this dependence we varied the parameters within their allowed range and found that the average mass changes at most by $\pm 30\%$, which shows that the result is rather robust. Although the value for the average mass we find with the mass moment method is marginally consistent with the result of the MACHO team, it definitely favours a lower average MACHO mass.

One can also consider other models with more general luminous and dark matter distributions, e.g. ones with a flattened halo or with anisotropy in velocity space⁶⁵, in which case the resulting value for the average mass would decrease significantly.

Another important quantity to be determined is the fraction f of the local dark mass density (the latter one given by ρ_0) detected in the form of MACHOs, which is given by $f \equiv M_\odot/\rho_0 \sim 126 \text{ pc}^3 \langle \mu^1 \rangle$. Using the values given by the MACHO collaboration for their two years data²⁹ (in particular $u_{TH} = 0.661$ corresponding

to $A > 1.75$ and an effective exposure $N_\star t_{obs}$ of $\sim 5 \times 10^6$ star-years for the observed range of the event duration T between ~ 20 - 50 days) we find $f \sim 0.54$, which compares quite well with the corresponding value ($f \sim 0.45$ based on the six events we consider) calculated by the MACHO group in a different way. The value for f is obtained again by assuming a standard spherical halo model.

Similarly, one can also get information from the events detected so far towards the galactic bulge. The mean MACHO mass, which one gets when considering the first eleven events detected by OGLE in the galactic bulge, is $\sim 0.29 M_\odot$ ⁶¹. From the 40 events discovered during the first year of operation by the MACHO team³⁶ (we considered only the events used by the MACHO team to infer the optical depth without the double lens event) we get an average value of $0.16 M_\odot$. Both values are obtained under the assumption that the lenses are located in the disk. For a more detailed analysis see ref⁶⁶. The lower value inferred from the MACHO data is due to the fact that the efficiency for the short duration events (\sim some days) is substantially higher for the MACHO experiment than for the OGLE one. These values for the average mass suggest that the lens are faint stars.

Once several moments $\langle \mu^m \rangle$ are known one can get information on the mass distribution $dn_0/d\mu$. Since at present only few events towards the LMC are at disposal the different moments (especially the higher ones) can be determined only approximately. Nevertheless, the results obtained so far are already of interest and it is clear that in a few years, due also to the new experiments under way (such as EROS II, OGLE II and MOA in addition to MACHO), it will be possible to draw more firm conclusions.

6 Formation of dark clusters

A major problem concerns the formation of MACHOs, as well as the nature of the remaining amount of dark matter in the galactic halo. We feel it hard to conceive a formation mechanism which transforms with 100% efficiency hydrogen and helium gas into MACHOs. Therefore, we expect that also cold clouds (mainly of H_2) should be present in the galactic halo. Recently, we have proposed a scenario^{67,68,69,70,71} in which dark clusters of MACHOs and cold molecular coulds naturally form in the halo at galactocentric distances larger than 10–20 kpc, with the relative abundance possibly depending on the distance. Similar scenario have also been considered in refs.^{72,73}.

The evolution of the primordial proto globular cluster clouds (which make up the proto-galaxy) is expected to be very different in the inner and outer parts of the Galaxy, depending on the decreasing ultraviolet flux (UV) from the centre as the galactocentric distance R increases. In fact, in the outer halo no substantial H_2 depletion should take place, owing to the distance suppression of the UV flux. Therefore, the clouds cool and fragment - the process stops when the fragment mass becomes $\sim 10^{-2} - 10^{-1} M_\odot$. In this way dark clusters should form, which contain brown dwarfs and also cold H_2 self-gravitating cloud, along with some residual diffuse gas (the amount of diffuse gas inside a dark cluster has to be low, for otherwise it would have been observed in the radio band).

We have also considered several observational tests for our model^{67,74}. In par-

ticular, a signature for the presence of molecular clouds in the galactic halo should be a γ -ray flux produced in the scattering of high-energy cosmic-ray protons on H_2 . As a matter of fact, an essential information is the knowledge of the cosmic ray flux in the halo. Unfortunately, this quantity is unknown and the only available information comes from theoretical considerations. Nevertheless, we can make an estimate of the expected γ -ray flux and the best chance to detect it is provided by observations at high galactic latitude. Accordingly, we find a γ -ray flux (for $E_\gamma > 100$ MeV) $\Phi_\gamma(90^\circ) \simeq \tilde{f} (0.4 - 1.8) \times 10^{-5}$ photons $\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$ (\tilde{f} stands for the fraction of halo dark matter in the form of gas), if the cosmic rays are confined in the galactic halo, otherwise, if they are confined in the local galaxy group ${}^{75} \Phi_\gamma(90^\circ) \simeq \tilde{f} (0.6 - 3) \times 10^{-7}$ photons $\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$. These values should be compared with the measured flux by the SAS-II satellite for the diffuse background of $(0.7 - 2.3) \times 10^{-5}$ photons $\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$ or the corresponding flux found by EGRET of $\sim 1.1 \times 10^{-5}$ photons $\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$. Thus, there is at present no contradiction with observations. Furthermore, an improvement of sensitivity for the next generation of γ -ray detectors will allow to clarify the origin of this flux or yield more stringent limits on \tilde{f} .

7 Conclusions

The mystery of the dark matter is still unsolved, however, thanks to the ongoing microlensing and pixel-lensing experiments there is hope that progress on its nature in the galactic halo can be achieved within the next few years. Substantial progress will also be done in the study of the structure of our Galaxy and this especially once data from the observations towards the spiral arms will be available. Microlensing is also very promising for the discovery of planets. Although being a rather young observational technique microlensing has already allowed to make substantial progress and the prospects for further contribution to solve important astrophysical problems look very bright.

It has also to be mentioned that it is well plausible that only a fraction of the halo dark matter is in form of MACHOs, either brown dwarfs or white dwarfs, in which case there is the problem of explaining the nature of the remaining dark matter and the formation of the MACHOs. Before invoking the need for new particles as galactic dark matter candidates for the remaining fraction, one should seriously consider the possibility that it is in the form of cold molecular clouds. A scenario this, for which several observational tests have been proposed, thanks to which it should be feasible in the near future to either detect or to put stringent limits on these clouds.

1. S.M. Faber and J.S. Gallagher, Ann. Rev. Astron. Astrophys. **17** (1979) 135
2. V. Trimble, Ann. Rev. Astron. Astrophys. **25** (1987) 425
3. J.H. Oort, Bull. Astron. Inst. Netherlands **6** (1932) 249
4. F. Zwicky, Helv. Phys. Acta **6** (1933) 110
5. V.C. Rubin and W.K. Ford, Astrophys. J. **159** (1970) 379
6. M. Persic, P. Salucci and F. Stel, Mont. Not. R. Astr. Soc. **281** (1996) 27

7. D. Zaritsky et al., *Astrophys. J.* **345** (1989) 759
8. D.N. Lin, B.F. Jones and A.R. Klemola, *Astrophys. J.* **439** (1995) 652
9. C.S. Kochanek, *Astrophys. J.* **457** (1996) 228
10. G. Jungman, M. Kamionkowski and K. Griest, *Phys. Rept.* **267** (1996) 195
11. B. Moore, *Nature* **370** (1994) 629
12. J.F. Navarro, C.S. Frenk and S.D. White, *Astrophys. J.* **462** (1996) 563
13. A. Burkert and J. Silk, *astro-ph* 9707343
14. C.J. Copi, D.N. Schramm and M.S. Turner, *Science* **267** (1995) 192
15. Particle Data Group, *Phys. Rev.* **D54** (1996) 109-111
16. F. De Paolis, G. Ingrosso, Ph. Jetzer and M. Roncadelli, *astro-ph* 9709052 to appear in *Astron. and Astrophys.*
17. B. Carr, *Annu. Rev. Astron. Astrophys.* **32** (1994) 531
18. A. De Rújula, Ph. Jetzer and E. Massó, *Astron. and Astrophys.* **254** (1992) 99
19. J. Bahcall, C. Flynn, A. Gould and S. Kirhakos, *Astrophys. J.* **435** (1994) L51
20. E. J. Kerins, *astro-ph* 9610070
21. E. J. Kerins, *astro-ph* 9704179
22. C.M. Tamanaha, J. Silk, M.A. Wood and D.E. Winget, *Astrophys. J.* **358** (1990) 164
23. S.D. Kawaler, *Astrophys. J.* **467** (1996) L61
24. B. Paczyński, *Astrophys. J.* **304** (1986) 1
25. B. Paczyński, *Astrophys. J.* **371** (1991) L63
26. Griest, K. et al., 1991, *Ap. J.* **372**, L79
27. E. Aubourg et al., *Nature* **365** (1993) 623
28. C. Alcock et al., *Nature* **365** (1993) 621; *Astrophys. J.* **445**, (1995) 133
29. C. Alcock et al., *astro-ph* 9606165
30. C. Renault et al., *Astron. and Astrophys.* **324** (1997) L69
31. C. Alcock et al., *astro-ph* 9708190
32. A. Udalski et al., *Acta Astron.* **43** (1993) 289
33. A. Udalski et al., *Acta Astron.* **44** (1994) 165
34. M. Kiraga and B. Paczyński, *Astrophys. J.* **430** (1994) 101
35. C. Alard, in *Proceedings of the 12th IAP Astrophysics Colloquium*, Editions Frontières (1997) 37
36. C. Alcock et al., *Astrophys. J.* **479** (1977) 119
37. C. Alcock et al., *Astrophys. J.* **454** (1995) L125
38. R. Ansari et al., *Astron. and Astrophys.* **324** (1997) 843.
39. A.P.S. Croots and A.B. Tomaney, *Astrophys. J.* **473** (1996) L87
40. A. Gould, *Astrophys. J.* **455** (1995) 44
41. F. Abe et al., in *Proceedings of the 12th IAP Astrophysics Colloquium*, Editions Frontières (1997) 75
42. M. Albrow et al., in *Proceedings of the IAU Symposium 173 - Astrophysical Applications of Gravitational Lensing* (Melbourne, Australia), C.S. Kochanek and J.N. Hewitt editors, Kluwer, Dordrecht (1996), page 227
43. Pratt et al., *astro-ph* 9508039
44. C. Alcock et al., *astro-ph* 9702199

45. S. Mao and B. Paczyński, *Astrophys. J.* **374** (1991) L37

46. A. Gould and A. Loeb, *Astrophys. J.* **396** (1992) 104

47. D. Bennett and S.H. Rhie, *astro-ph* 9603158

48. S. Refsdal, *Mont. Not. R. Astr. Soc.* **134** (1966) 315

49. A. Gould, *Astrophys. J.* **421** (1994) L75x

50. B. Paczynski, *Annu. Rev. Astron. Astrophys.* **34** (1996) 419

51. E. Roulet and S. Mollerach, *Phys. Rept.* **279** (1997) 67

52. M. Dominik, Thesis University of Dortmund (1996)

53. A. De Rújula, Ph. Jetzer and E. Massó, *Mont. Not. R. Astr. Soc.* **250** (1991) 348

54. Ph. Jetzer, *Atti del Colloquio di Matematica (CERFIM)* **7** (1991) 259

55. K. Griest, *Astrophys. J.* **366** (1991) 412

56. A.P. Crotts, *Astrophys. J.* **399** (1992) L43

57. P. Baillon, A. Bouquet, Y. Giraud-Héraud and J. Kaplan, *Astron. and Astrophys.* **277** (1993) 1;

58. Ph. Jetzer, *Astron. and Astrophys.* **286** (1994) 426

59. Bahcall, J.N., & Soneira, R.M., 1980, *Ap. JS.* **44**, 73.

60. Mihalas, D., & Binney, J., 1981, *Galactic Astronomy* (San Francisco: W.H. Freeman and Co.).

61. Ph. Jetzer, *Astrophys. J.* **432** (1994) L43

62. Ph. Jetzer and E. Massó, *Phys. Lett. B* **323** (1994) 347

63. Ph. Jetzer and E. Massó, in the proceedings of the second Rome workshop: "The dark side of the Universe: experimental efforts and theoretical frameworks" (World Scientific, Singapore) (1996) 31

64. Ph. Jetzer, *Helv. Phys. Acta* **69**, 179 (1996)

65. F. De Paolis, G. Ingrosso and Ph. Jetzer, *Astrophys. J.* **470**, 493 (1996)

66. L. Grenacher, Diploma thesis University of Zürich (1997)

67. F. De Paolis, G. Ingrosso, Ph. Jetzer and M. Roncadelli, *Phys. Rev Lett.* **74**, 14 (1995)

68. F. De Paolis, G. Ingrosso, Ph. Jetzer and M. Roncadelli, *Astron. and Astrophys.* **295**, 567 (1995)

69. F. De Paolis, G. Ingrosso, Ph. Jetzer and M. Roncadelli, *Comments on Astrophys.* **18**, 87 (1995)

70. F. De Paolis, G. Ingrosso, Ph. Jetzer and M. Roncadelli, *Astrophys. and Space Science* **235**, 329 (1996)

71. F. De Paolis, G. Ingrosso, Ph. Jetzer and M. Roncadelli, *Int. J. Mod. Phys. D* **5**, 151 (1996)

72. D. Pfenniger, F. Combes and L. Martinet, *Astron. and Astrophys.* **285** (1994) 79

73. O.E. Gerhard and J. Silk, *Astrophys. J.* **472** (1996) 34

74. F. De Paolis, G. Ingrosso, Ph. Jetzer, A. Qadir and M. Roncadelli, *Astron. and Astrophys.* **299**, 647 (1995)

75. V.S. Berezinsky, P. Blasi and V.S. Ptuskin, *astro-ph* 9609048